Analytical and numerical evidence of the cascade reversal due to electron inertia

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Institute of Technology

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Outline

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1. Introduction

2. Results from analyzing 2D XMHD model

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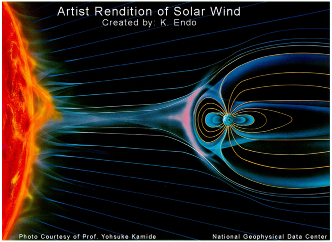

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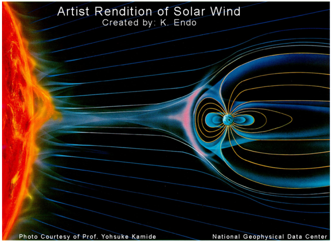

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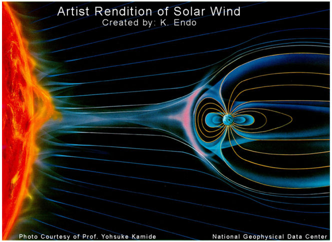

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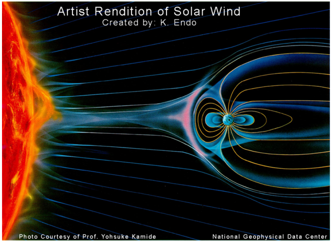

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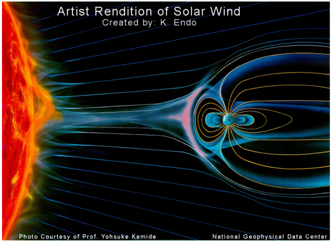
Artist Rendition of Solar Wind
Created by: K. Endo
Photo Courtesy of Prof. Yohsuke Kamide
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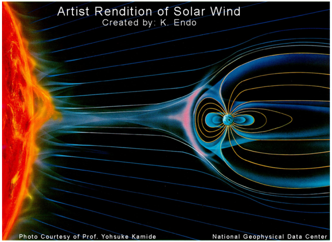

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$\omega := \nabla^2\phi$

$$\frac{\partial\psi^\star}{\partial t} = -[\phi,\psi^\star] \quad \text{and} \quad \frac{\partial\omega}{\partial t} = -[\phi,\omega] - [\nabla^2\psi,\psi^\star], \tag{2}$$

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- In this case we only have two quadratic Casimir Invariants[3].

$$G = \int d^2x\,\omega\psi^\star, \quad F = \frac{1}{2}\int d^2x\,(\psi^\star)^2 \tag{3}$$

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[3] T. J. Schep et al., "Generalized twofluid theory of nonlinear magnetic structures", Physics of Plasmas 1, 2843–2852 (1994)

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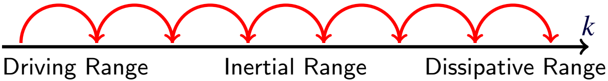

Figure: Schematics of the R-K Direct Cascade (DC) from large to small scales.

- The cascade direction can be determined from the AES the turbulence would relax to, if not for the input of energy [4]

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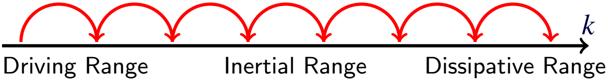

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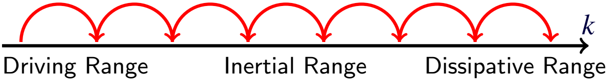

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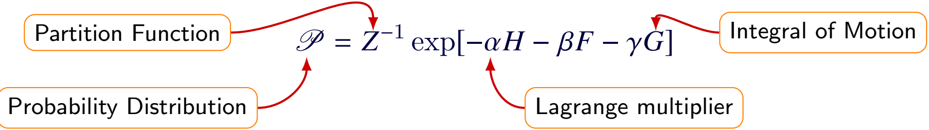

Partition Function

Integral of Motion

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Probability Distribution

Lagrange multiplier

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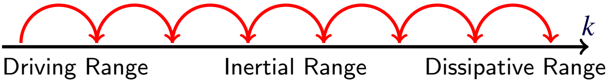

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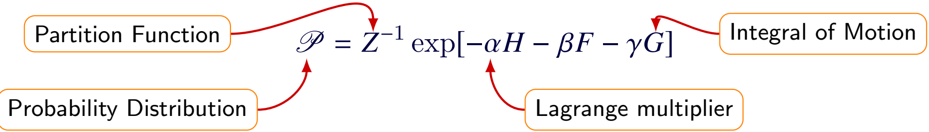

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- The approach has been invoked in HD [5] and MHD [6] turbulence

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[4] D. Biskamp, *Magnetohydrodynamic Turbulence*, (July 2003), p. 310

[5] R. H. Kraichnan, "Inertial Ranges in Two-Dimensional Turbulence", Physics of Fluids **10**, 1417–1423 (1967)

[6] U. Frisch et al., "Possibility of an inverse cascade of magnetic helicity in magnetohydrodynamic turbulence", Journal of Fluid Mechanics **68**, 769–778 (1975)

Spectral scaling of helicity across the scales

- Performing Fourier expansion, e.g. $\psi^\star(x) = \sum_k \psi^\star_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$ for Casimirs:

$$F := \frac{1}{2}\sum_{\boldsymbol{k}} |\psi^\star_{\boldsymbol{k}}|^2, \quad \mathcal{H} = \frac{1}{2}\sum_{\boldsymbol{k}} \left(\frac{|\omega_{\boldsymbol{k}}|^2}{k^2} + \frac{k^2|\psi^\star_{\boldsymbol{k}}|^2}{1 + k^2 d_e^2} \right), \ldots$$

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- MHD limit: $kd_e \ll 1 \Rightarrow \boxed{2\pi k\langle F\rangle \approx \mathcal{O}\left(k^{-1}\right),} \quad 2\pi k\langle\mathcal{H}\rangle \approx \mathcal{O}(k)$

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[7] D. Fyfe et al., "Magnetic dynamo action in two-dimensional turbulent magneto-hydrodynamics", Journal of Plasma Physics **17**, 317–335 (1977)

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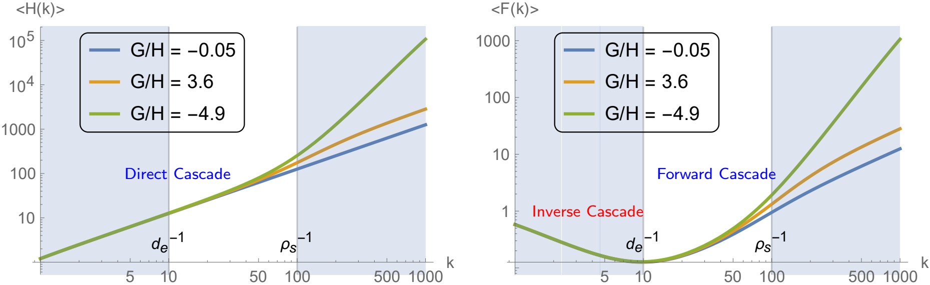

(a) Energy (H) (b) Remnant Helicity (F)

Figure: The parameters used here are $\alpha = 10$, $\beta = 1$ and $\gamma = \{0, -.75, 1\}$ is varied so that different values of G are obtained (color-coded).

- We predict a cascade reversal of F at the electron skin depth [8]

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[8] G Miloshevich et al., "Direction of cascades in a magnetofluid model with electron skin depth and ion sound larmor radius scales", version 1, (2018).

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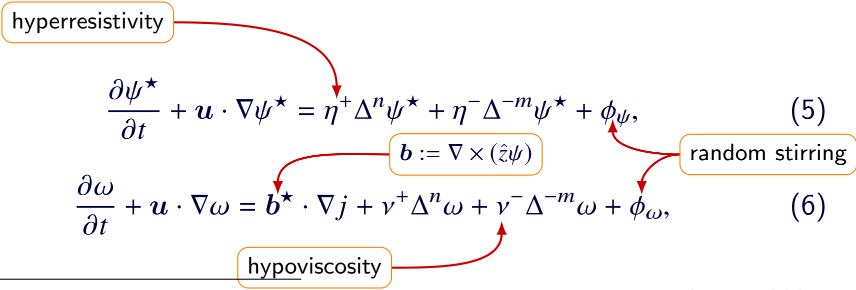

hyperresistivity

$$\frac{\partial \psi^\star}{\partial t} + \boldsymbol{u} \cdot \nabla \psi^\star = \eta^+ \Delta^n \psi^\star + \eta^- \Delta^{-m} \psi^\star + \phi_\psi, \tag{5}$$

$\boldsymbol{b} := \nabla \times (\hat{z}\psi)$

random stirring

$$\frac{\partial \omega}{\partial t} + \boldsymbol{u} \cdot \nabla \omega = \boldsymbol{b}^\star \cdot \nabla j + \nu^+ \Delta^n \omega + \nu^- \Delta^{-m} \omega + \phi_\omega, \tag{6}$$

hypoviscosity

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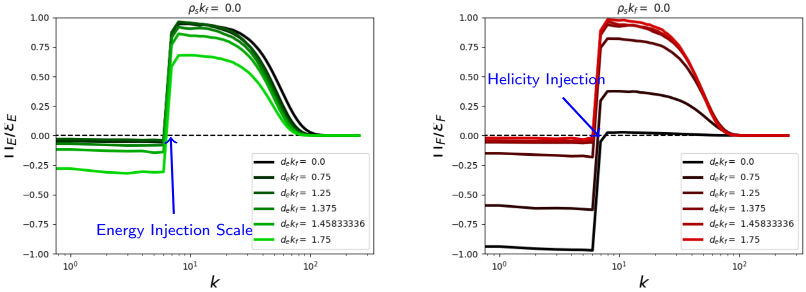

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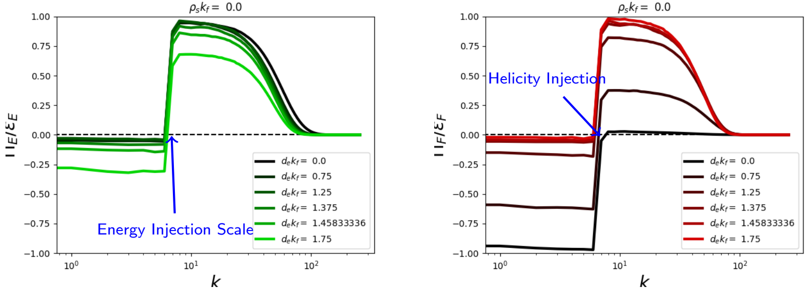

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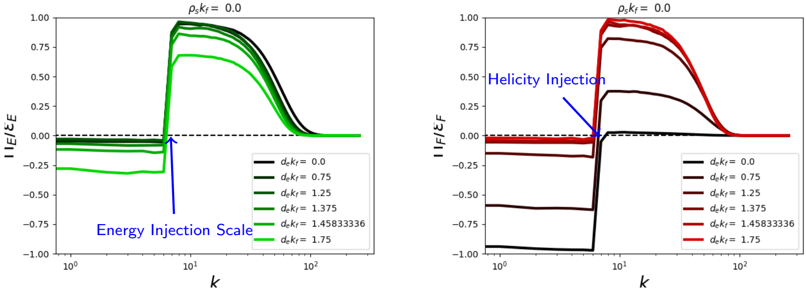

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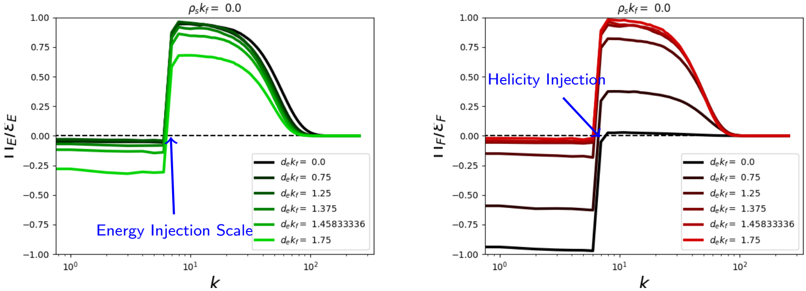

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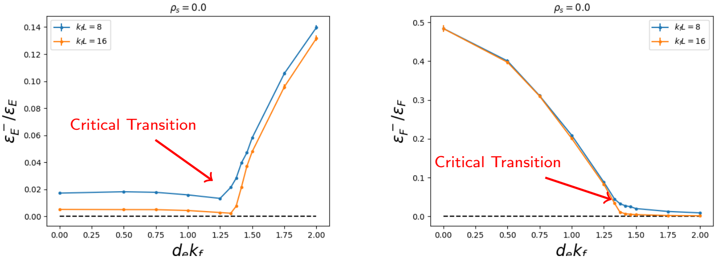

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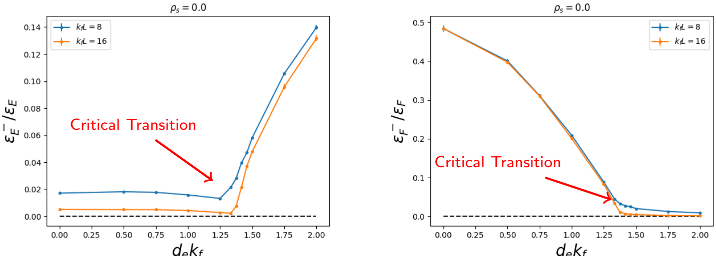

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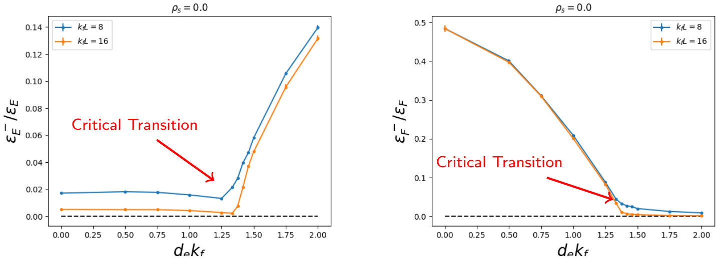

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- This type of transition has been observed in $2D \rightarrow 3D$ fluids[10].

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[10] S. J. Benavides and A. Alexakis, "Critical transitions in thin layer turbulence", Journal of Fluid Mechanics **822**, 364385 (2017)

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Models	2D Analytics	2D Numerics	3D XMHD
Energy	Forward Cascade	Dual	Forward Cascade
Helicity	Reversal	Reversal	Reversal

Summing up

- We studied AES taking microscales electron inertia into account.
- We predicted cascade reversal on short scales due to d_e[11]
- Numerics demonstrate this, but Energy dual cascades.

Models	2D Analytics	2D Numerics	3D XMHD
Energy	Forward Cascade	Dual	Forward Cascade
Helicity	Reversal	Reversal	Reversal

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[11] G. Miloshevich et al., "On the structure and statistical theory of turbulence of extended magnetohydrodynamics", New Journal of Physics **19**, 015007, 015007 (2017)

Summing up

- We studied AES taking microscales electron inertia into account.
- We predicted cascade reversal on short scales due to d_e[11]
- Numerics demonstrate this, but Energy dual cascades.

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- The lack of IC in 3DXMHD at $k \leq d_e^{-1}$ may influence dynamo [12]

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[11] G. Miloshevich et al., "On the structure and statistical theory of turbulence of extended magnetohydrodynamics", New Journal of Physics **19**, 015007, 015007 (2017)

[12] A. Alexakis et al., "On the inverse cascade of magnetic helicity", The Astrophysical Journal **640**, 335 (2006)

Future work

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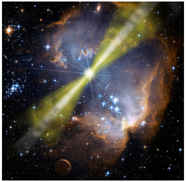

Figure: Artist's illustration of a GRB occurring in a star-forming region. Energy is beamed into two narrow jets.

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[13] Y. Kawazura et al., "Action principles for relativistic extended magnetohydrodynamics: A unified theory of magnetofluid models", Physics of Plasmas **24**, 022103, 022103 (2017)

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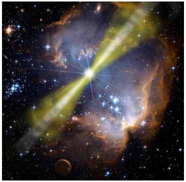
- New Hamiltonian Relativistic XMHD may allow us to seek Casimirs[13]



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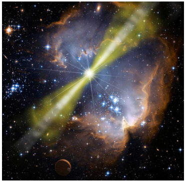

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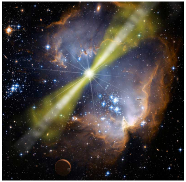
- We hope the astrophysics/plasma community will find our work useful



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